Mobilization of Oil Ganglia

Following secondary recovery processes in conventional light oil reservoirs, more than half the original oil in place may remain trapped as a discontinuous phase. During the previous recovery processes these oil ganglia have been pinched off by capillary forces and remain immobile while the continuous phase which surrounds them is able to flow freely. Furthermore if a portion of this oil is mobilized in a tertiary recovery process the conditions required to apply Darcy's equation to the flow of either phase are violated. These are also problems which are encountered during *in-situ* recovery techniques in tar sands where the mobilization of the heavy oil occurs as a discontinuous phase. In this paper the relevant flow equations are derived. Also a parameter is deduced which directly determines the criterion for mobilization.

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SCOPE

Following the immiscible displacement of oil by water from a water wet porous medium, a residual saturation of oil remains in the pore spaces. Here isolated oil ganglia occupying a single or a number of connected pore spaces are pinched off by capillary forces, and remain immobile even when water flooding continues at a given rate. In enhanced oil recovery and for many recovery techniques in the *in-situ* extraction of heavy oil, the mobilization and flow of such droplets is of fundamental importance to the processes being considered.

The mechanism by which oil ganglia is mobilized or trapped has therefore been the subject of much research (Melrose and Brandner, 1974; Larson et al., 1977; Ng et al., 1980; Pathak et al., 1980; Payatakes et al., 1980; Clampitt and Fleming, 1980). A relationship which has been sought after in particular is the specification of the residual oil saturation in terms of wettability, flow rate, viscosity, interfacial tension, and gross medium characteristics. Laboratory tests have shown a correlation between S_{or}, the residual oil saturation, and the capillary number $(\mu_1 q_1/\alpha)$ (Ng et al., 1978). For small-enough capillary number, say $\leq 10^{-6}$, the residual saturation assumes a value which is believed to be determined primarily by the nature of the porous medium. The correlation generally found is that for increasing N_c, the residual saturation decreases roughly logarithmically, reaching virtually zero for $N_c \sim 10^{-2}$.

The underlying physical mechanism of oil mobilization is believed (Larson et al., 1977) to hinge on whether it is possible to satisfy the (static) Laplace equation. This equation specifies the pressure exerted by an interfacial element as a function of the local curvature. Constraints on the value of the local curvature are then imposed by the solid matrix and as a result the interface is able to react only to a limited range of droplet pressures. Then when the interfacial tension can no longer balance the variation in normal tractions on the blob, it must be mobilized. This concept in conjunction with Darcy's law and dimensional considerations leads to (Ng et al., 1978)

$$\ell = \frac{\beta k_e}{r}\frac{1}{N_c},$$

a critical length for the blob below which it will be stranded. Here k_e is the effective permeability for water, r is a characteristic length of the medium, and β is a dimensionless (undetermined) coefficient. The sum of the volumes of all these shorter blobs in a unit volume divided by the porosity is then the new residual oil saturation. In this way a correlation can be established between S_{or} and N_c. Recently Larson et al. (1977) made use of percolation theory to establish a (model-dependent) distribution function for blob length and thus were able to approach the $S_{or} - N_c$ correlation from the theoretical side. A comparison of this theory with empirical results appears encouraging.

These previous considerations, however, apply only to steady states in which the discontinuous phase is at rest. In this paper we wish to present a dynamic study permitting a nonvanishing $\bar{q}_2$ (i.e., a partially mobilized discontinuous phase). Although the ratio $\mu_1 \bar{q}_1/\alpha$ plays a fundamental role, much still remains to be learned about blob mobilization from such dynamic studies.

CONCLUSIONS AND SIGNIFICANCE

This analysis of the mobilization and flow of a discontinuous nonwetting phase has produced a number of important results. The first is a generalization, Eq. 32, of the standard Darcy equation to include the effects of moving droplets on the flow of a continuous fluid phase in a porous medium. Also, a second flow Eq. 47 is derived describing the average flow rate per unit volume of a partially mobilized, discontinuous, nonwetting phase. The first flow equation is completely consistent with Darcy's equation for single-phase flow in the limit as the flow of the discontinuous phase approaches zero. In this limit the second flow equation then yields a criterion (Eq. 48) for an immobile discontinuous phase. The violation of this condition then is interpreted as the criterion for the mobilization of the discontinuous phase.

Further, the violation of Darcy's equation for the continuous phase as illustrated here, when one has mobilized droplets, brings into question the application of Darcy's equation (and relative permeability concepts) in the frontal region during immiscible displacement. In particular, the case of water displacing oil from a water wet porous medium, which is completely coated by a layer of connate water, may be considered as a special case of the equations obtained in this paper. One thus obtains a particular example of a frontal displacement where Darcy's equation is clearly violated. Attempts to salvage this theory using relative permeability concepts are questionable, since the extension required is an incorporation of information about the averaged effects of the interfaces between the fluids.

VOLUME AVERAGING

We approach the problem of flow and stability by starting at the microscopic level (i.e., the pore scale). We then systematically apply an averaging procedure to filter out the overabundance of microscopic details, while keeping the basic microscopic origins of the relevant macroscopic quantities firmly in sight. We choose here to work with volume averaging rather than ensemble averaging to be able to always talk in terms of real interfaces (Hubbard, 1956; Whittaker, 1966, 1969; Slattery, 1967, 1969, 1970; Patel et al., 1972; Newman, 1977).

Let V be regions in the porous medium, defined to be of identical shapes, volumes and orientations and "centered" at points $\vec{x}$ of the medium. For example, if V were chosen to be spheres, it is simplest to assign each V to its center $\vec{x}$.

Let $f_1(\vec{x})$ be a physical quantity associated with fluid 1, e.g., its mass density. We define $f_1(\vec{x})$ to be zero everywhere outside fluid 1. The volume average of f_1 over any volume V is defined as

$$\langle f_1 \rangle = \frac{1}{V} \int_V f_1(\vec{x}) dV. \tag{1}$$

The quantity $\langle f_1 \rangle$, regarded as a function of the "centroids" of V, is a smooth function, provided the value V is large compared with the grain sizes.

A related quantity, $\bar{f}_1$, is defined as

$$\bar{f}_1 = \frac{1}{V_1} \int_V f_1(\vec{x}) dV \tag{2}$$

where V_1 is the volume of fluid 1 in V. The fractional volume of fluid 1 is

$$\begin{aligned} \eta_1 &= V_1/V \\ &= \eta S_1 \end{aligned} \tag{3}$$

where η is the porosity and S_1 is the saturation. Equation 2 may be rewritten as

$$\bar{f}_1 = \frac{1}{\eta_1} \langle f_1 \rangle. \tag{4}$$

The averaging procedure is based on the following two averaging theorems, which link the averages of derivatives to derivatives of averages:

$$\int_V \partial_i f_1 dV = \partial_i \int_V f_1 dV + \int_{A_1} f_1 n_i dA, \quad i = x,y,z \tag{5}$$

$$\int_V \partial_t f_1 dV = \partial_t \int_V f_1 dV - \int_{A_1} f_1 \vec{u} \cdot \vec{n} dA. \tag{6}$$

Here A_1 refers to all interfaces (in V) involving fluid 1. Thus, for n-phase flow, $A_1 = A_{12} + \ldots + A_{1n} + A_{1s}$, where the subscript s refers to the solid. The convention is chosen such than $\vec{n}$ is directed outward from fluid 1. $\vec{u}$ are the velocities of the interfaces. Equation 5 is the multiphase version of what is sometimes known as the Whitaker-Slattery averaging theorem (Slattery, 1969; Whitaker, 1969). Equations 5 and 6 can be proved using ordinary analysis without undue difficulties.

TWO-PHASE FLOW

The microscopic dynamical equation for an incompressible fluid is (using the summation convention)

$$\frac{\partial}{\partial t}(\rho v_i) + \partial_k \Pi_{ik} = \rho g_i \tag{7}$$

where

$$\Pi_{ik} = p\delta_{ik} + \rho v_i v_k - \sigma_{ik} \tag{8}$$

and

$$\sigma_{ik} = \mu(\partial_i v_k + \partial_k v_i). \tag{9}$$

In the following we shall assume the validity of no-slip condition. Now operating with $1/V \int_V dV$ on Eq. 7 for fluid 1, and applying the two averaging theorems we obtain

$$\begin{aligned} 0 &= \frac{1}{V}\int \frac{\partial}{\partial t}(\rho_1 v_i^{(1)}) dV + \frac{1}{V}\int \partial_k \Pi_{ik}^{(1)} dV - \frac{1}{V}\int \rho_1 g_i dV \\ &= \frac{\partial}{\partial t}(\eta_1 \rho_1 \bar{v}_i^{(1)}) + \partial_k\left[\eta_1 \bar{p}_1 \delta_{ik} + \eta_1 \rho_1 \overline{v_i^{(1)} v_k^{(1)}} - \eta_1 \bar{\sigma}_{ik}^{(1)}\right] \\ &\quad + \frac{1}{V}\int_{A_1} (p_1 \delta_{ik} - \sigma_{ik}^{(1)}) n_k dA - \eta_1 \rho_1 g_i \end{aligned} \tag{10}$$

where

$$\begin{aligned} \eta_1 \bar{\sigma}_{ik}^{(1)} &= \frac{1}{V}\int_V \sigma_{ik}^{(1)} dV \\ &= \mu_1\left[\partial_i q_k^{(1)} + \partial_k q_i^{(1)} + \frac{1}{V}\int_{A_1} (v_k^{(1)} n_i + v_i^{(1)} n_k) dA\right]. \end{aligned} \tag{11}$$

Here

$$\vec{q}^{(1)} = \frac{1}{V}\int_V \vec{v}^{(1)} dV = \eta_1 \bar{\vec{v}}^{(1)} \tag{12}$$

is the filter velocity. The condition of incompressibility

$$\partial_i v_i^{(1)} = 0 \tag{13}$$

can be averaged to read

$$\begin{aligned} 0 &= \frac{1}{V}\int_V \partial_i v_i^{(1)} dV \\ &= \partial_i q_i^{(1)} + \frac{1}{V}\int_{A_{12}} v_i^{(1)} n_i dA \end{aligned}$$

i.e.

$$\nabla \cdot \vec{q}^{(1)} = -\frac{1}{V}\int_{A_{12}} \vec{v}^{(1)} \cdot \vec{n} dA. \tag{14}$$

Alternatively, we may find from the equation of continuity

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 \vec{v}^{(1)}) = 0$$

that

$$\begin{aligned} 0 &= \frac{1}{V}\int_V \left[\frac{\partial \rho_1}{\partial t} + \nabla \cdot \rho_1 \vec{v}^{(1)}\right] dV \\ &= \frac{1}{V}\left\{\frac{\partial}{\partial t}\int \rho_1 dV - \int_{A_1} \rho_1 \vec{v}_1 \cdot \vec{n} dA \right. \\ &\quad \left. + \nabla \cdot \int \rho_1 \vec{v}^{(1)} dV + \int_{A_1} \rho_1 \vec{v}^{(1)} \cdot \vec{n} dA\right\} \\ &= \rho_1\left[\frac{\partial \eta_1}{\partial t} + \nabla \cdot \vec{q}^{(1)}\right] \end{aligned}$$

and thus

$$\frac{\partial \eta_1}{\partial t} + \nabla \cdot \vec{q}^{(1)} = 0. \tag{15}$$

Entirely similar equations can, of course, be written down at once for fluid 2.

The flow Eq. 10 describes a fluid of density $\eta_1 \rho_1$, velocity $\bar{\vec{v}}_1$, with momentum flux

$$\eta_1 \bar{p}_1 \delta_{ik} + \eta_1 \rho_1 \overline{v_i^{(1)} v_k^{(1)}} - \eta_1 \bar{\sigma}_{ik}^{(1)}, \tag{16}$$

and acted upon by a body force (per unit volume)

$$\frac{1}{V}\int_{A_1} (\sigma_{ik}^{(1)} - p_1 \delta_{ik}) n_k dA + \rho_1 \eta_1 g_i. \tag{17}$$

This interpretation is consistent with the equation of continuity (Eq. 15) in the form

$$\frac{\partial \eta_1 \rho_1}{\partial t} + \nabla \cdot (\eta_1 \rho_1 \bar{\vec{v}}_1) = 0. \tag{18}$$

Since $\bar{\vec{v}}_1 = \vec{q}_1/\eta_1 = (\eta/s_1)\vec{q}_1$, it is an observable quantity, and should be just as acceptable as $\vec{q}_1$ itself. Moreover, no equation of continuity can be written for a hypothetical fluid of density ρ_1 (= const) and velocity $\vec{q}_1$, except when $\partial \eta_1/\partial t = 0$. In a displacement process let the velocity of the flood front be $\sim v$ and its thickness $\sim \ell$. Then

$$\frac{\partial \eta_1}{\partial t} \sim \frac{v}{\ell} \tag{19}$$

which may not be small for a narrow front.

TWO-PHASE DARCY FLOW

In the static limit, Eq. 10 becomes

$$\partial_i(\eta_1 \bar{p}_1) + \frac{1}{V}\int_{A_1} p_1 dA_i - \eta_1 \rho_1 g_i = 0. \tag{20}$$

The Navier-Stokes equation can be solved to give

$$p_1 = p_o + \rho_1 g_i x_i \tag{21}$$

where, assuming fluid 1 forms a continuous body, p_o is constant throughout. Hence

$$\langle p_1 \rangle \equiv \eta_1 \bar{p}_1 = \frac{1}{V}\int_V p_1 dV = \frac{1}{V}\int (p_o + \rho_1 g_i x_i) dV$$
$$= \eta_1 (p_o + \rho_1 g_i x_i). \tag{22}$$

Equations 20 and 21 together imply that, in the static limit,

$$\frac{1}{V}\int_{A_1} p_1 dA_i = -\partial_i(\eta_1 \bar{p}_1) + \eta_1 \rho_1 g_i$$
$$= -\bar{p}_1 \partial_i \eta_1 - \eta_1 \partial_i \bar{p}_1 + \eta_1 \rho_1 g_i$$
$$= -\bar{p}_1 \partial_i \eta_1 - \eta_1 \partial_i [p_o + \rho_1 g_j x_j] + \eta_1 \rho_1 g_i$$
$$= -\bar{p}_1 \partial_i \eta_1. \tag{23}$$

For slow flow cases, the averaged flow Eq. 10 reads

$$\partial_i(\eta_1 \bar{p}_1) + \frac{1}{V}\int_{A_1} p_1 dA_i - \frac{1}{V}\int_{A_1} \sigma_{ik}^{(1)} dA_k - \eta_1 \rho_1 g_i = 0, \tag{24}$$

where we assumed we can ignore the inertial terms (including the term quadratic in $v^{(1)}$) and the averaged viscous term

$$\partial_k(\eta_1 \sigma_{ik}^{(1)}),$$

which is a sum of derivatives of the velocity, and which reduces to the Brinkman term in the limit of one-phase flow.

For the term $1/V \int p_1 dA_i$ in Eq. 24, we may write it as

$$\frac{1}{V}\int_{A_1} p_1 dA_i = \frac{1}{V}\int (p_1 - \bar{p}_1) dA_i + \frac{1}{V}\int \bar{p}_1 dA_i$$
$$= \frac{1}{V}\int_{A_1} (p_1 - \bar{p}_1) dA_i - \bar{p}_1 \partial_i \eta_1, \tag{25}$$

since $1/V \int_{A_1} n_i dA = -\partial_i \eta_1$ according to the Whitaker-Slattery theorem (Eq. 5). Now substituting Eq. 25 into Eq. 24 we find

$$-\frac{1}{V}\int_{A_1} (p_1 - \bar{p}_1) dA_i + \frac{1}{V}\int_{A_1} \sigma_{ik}^{(1)} dA_k = \eta_1(\partial_i \bar{p}_1 - \rho_1 g_i) \tag{26}$$

which expresses the balance of four forces, namely, the gradient of average pressure, the gravitational force, the shear force, and the imbalance between the average pressure and the microscopic pressure of fluid 1, summed over the interface between the fluids. The shear force term can be decomposed thus

$$\frac{1}{V}\int_{A_1} \sigma_{ik}^{(1)} dA_k = \frac{1}{V}\int_{A_{1s}} \sigma_{ik}^{(1)} dA_k + \frac{1}{V}\int_{A_{12}} \sigma_{ik}^{(1)} dA_k. \tag{27}$$

Expanding in powers of the velocities and keeping only the lowest order terms, we have

$$\frac{1}{V}\int_{A_{1s}} \sigma_{ik}^{(1)} dA_k = -\mu_1 a_1 \bar{v}_i^{(1)} \tag{28}$$

$$\frac{1}{V}\int_{A_{12}} \sigma_{ik}^{(1)} dA_k = -\mu_1 b_1 (\bar{v}_i^{(1)} - \bar{v}_i^{(2)}) \tag{29}$$

where a_1 and b_1 depend on the structure of the porous medium, the saturation and the relative wettability of the fluids.

On the RHS of Eq. 28 $\bar{v}_i^{(1)}$ is the velocity of fluid 1 relative to the medium, which is taken to be stationary. Clearly, it is the relative velocity that gives rise to the force. Similarly Eq. 29 involves as a first approximation the relative velocity of the two fluids, rather than just $\bar{v}^{(1)}$ alone.

When there is flow, the microscopic p_1 appears to deviate from $\bar{p}_1$ by a term proportional to μ_1, as the simplest cases of Stokes flow indicate. Thus we may suppose

$$-\frac{1}{V}\int_{A_{1s}} (p_1 - \bar{p}_1) dA_i = -\mu_1 c_1 \bar{v}_i^{(1)} \tag{30}$$

and

$$-\frac{1}{V}\int_{A_{12}} (p_1 - \bar{p}_1) dA_i = -\mu_1 d_1 (\bar{v}_i^{(1)} - \bar{v}_i^{(2)}) \tag{31}$$

in addition to Eqs. 28 and 29. These four equations define the parameters a_1, b_1, c_1, d_1 and give them their physical meanings.

Upon substituting the above relations into Eq. 26, the flow equation

$$\mu_1 \left[\frac{a_1 + b_1 + c_1 + d_1}{\eta_1} q_i^{(1)} - \frac{b_1 + d_1}{\eta_2} q_i^{(2)} \right] = -\eta_1(\partial_i \bar{p}_1 - \rho_1 g_i) \tag{32}$$

is obtained.

This is our flow equation for fluid 1. Note that the parameters occur only in the combinations $a_1 + c_1$ and $b + d_1$. It reduces to the conventional Darcy's equation if $(b_1 + d_1)\vec{q}^{(2)}/\eta_2$ can be neglected. This will happen if fluid 2 is immobile, i.e., $q^{(2)} \rightarrow 0$, or if fluid 2 is of vanishing saturation. Another circumstance in which Darcy's equation holds is when $\vec{q}^{(2)}$ is strictly proportional to $\vec{q}^{(1)}$.

For an immobile fluid 2, Eq. 32 reduces to

$$\vec{q}^{(1)} = -\frac{\eta_1^2}{a_1 + b_1 + c_1 + d_1} \cdot \frac{1}{\mu_1} (\nabla \bar{p}_1 - \rho_1 \vec{g}) \tag{33}$$

and hence

$$kk_1 = \frac{\eta_1^2}{a_1 + b_1 + c_1 + d_1} \tag{34}$$

is the effective permeability.

An equation similar to Eq. 32 can of course also be written down for fluid 2,

$$\mu_2 \left[\frac{a_2 + b_2 + c_2 + d_2}{\eta_2} \vec{q}^{(2)} - \frac{b_2 + d_2}{\eta_1} \vec{q}^{(1)} \right] = -\eta_2 [\nabla \bar{p}_2 - \rho_2 \vec{g}] \tag{35}$$

provided the fluid also forms a continuous body. The parameters $a_2, \ldots, d_2$ are defined by equations similar to Eqs. 25 to 31. If fluid 2 is not continuous, then in the static solution

$$p_2 = p_b + \rho_2 \vec{g} \cdot \vec{x}. \tag{36}$$

p_b is constant only throughout the "blob" b, the average pressure $\bar{p}_2$ behaves "erratically," and the arguments leading to Eqs. 30 and 31 can no longer be upheld.

It will be noted that if $q^{(2)} = 0$, according to Eq. 35

$$-\frac{\mu_2(b_2 + d_2)}{\eta_1} \vec{q}^{(1)} = -\eta_2 [\nabla \bar{p}_2 - \rho_2 \vec{g}]. \tag{37}$$

The flow of fluid 1 exerts a dragging force on fluid 2, which must be countered with pressure gradient and gravitational force.

MOBILIZATION CRITERIA FOR NONWETTING DROPLETS

It is generally agreed that capillary phenomena play an important role in multiphase flows in a porous medium, due to the relatively large curvatures observed for the interfaces.

To render the conventional system of equations complete, a macroscopic capillary pressure $\bar{p}_c$ is usually introduced through the equation

$$\bar{p}_2 - \bar{p}_1 = \bar{p}_c(S_1) \tag{38}$$

which is a direct analogue of the static Laplace equation

$$p_2 - p_1 = p_c \equiv \alpha C \tag{39}$$

where C is the curvature of the interface.

For strictly static cases in a uniform field, the pressures in the interiors are trivially related to those at the boundary; therefore, knowing $p_2 - p_1$ at the boundary allows one to determine $p_2 - p_1$ everywhere. If these conditions cannot be assumed, it seems that such a simple-minded relationship as Eq. 38 must in general break down (Spanos, 1979).

The concept of effective interfacial tension α^* has been introduced in the literature in connection with the phenomenon of fingering (Chuoke et al., 1959). The most probable finger sizes are determined by this quantity; therein lies the importance of α^*. While there are some vague experimental supports for the existence of α^*, theoretical understanding is still far from adequate.

In the usual scenario, the system is divided into two regions which meet at the flood front. In each region only one fluid is flowing. The macroscopic curvature C^* of the front supposedly introduces a discontinuity in the pressures of the displacing and displaced fluid across the front, given by

$$p_2|_+ - p_1|_- = \alpha^* C^* \tag{40}$$

in complete analogy to the Laplace equation.

In reality the only way interfacial tension can make its entrance in a dynamic situation is through the boundary condition

$$p_2 n_i - \sigma_{ij}^{(2)} n_j = p_1 n_i - \sigma_{ij}^{(1)} n_j + \alpha C n_i \tag{41}$$

on A_{12}. The sign convention on C is such that it is positive if the centers of curvature are in fluid 2. Examining the flow Eq. 10 and its fluid 2 analogue, we observe that the interfacial tension effects enter naturally if these two equations are summed. We thus obtain

$$\frac{\partial}{\partial t}\left(\sum_{\alpha=1}^{2} \rho_\alpha q_i^\alpha\right) + \partial_k\left[\sum_\alpha \eta_\alpha \bar{p}_\alpha \delta_{ik} + \eta_\alpha \rho_\alpha \overline{v_i^\alpha v_k^\alpha} - \eta_\alpha \bar{\sigma}_{ik}^\alpha\right] + \frac{1}{V}\sum_\alpha \int_{A_{\alpha s}} (p_\alpha \delta_{ik} - \sigma_{ik}^\alpha) dA_k - \sum_\alpha \eta_\alpha \rho_\alpha g_i - \frac{1}{V}\int_{A_{12}} \alpha C n_i dA = 0. \tag{42}$$

Neglecting the inertial terms and the "Brinkman term" (viscous dissipation within the fluids) from Eq. 42 we obtain with the aid of the identities

$$\partial_i \eta_\alpha + \frac{1}{V}\int_{A_\alpha} n_i dA = 0, \qquad \alpha = 1,2,$$

the following equation,

$$0 = \frac{1}{V}\int_{A_{1s}} (p_1 - \bar{p}_1) n_i dA + \frac{1}{V}(\bar{p}_2 - \bar{p}_1)\int_{A_{12}} n_i dA + \eta_1 \partial_i \bar{p}_1 + \eta_2 \partial_i \bar{p}_2 - \frac{1}{V}\int_{A_{1s}} \sigma_{ik}^{(1)} dA_k - (\eta_1\rho_1 + \eta_2\rho_2) g_i - \frac{\alpha}{V}\int_{A_{12}} C n_i dA. \tag{43}$$

Here we have further assumed that no oil-sand contact exists, i.e., $A_{2s} = 0$. The two integrals over A_{1s} can be eliminated using Eqs. 28 and 30, so that Eq. 43 becomes

$$0 = \mu_1 \frac{a_1 + c_1}{\eta_1} q_i^{(1)} + \eta_1 \partial_i \bar{p}_1 - \eta_1 \rho_1 g_i + \eta_2 \partial_i \bar{p}_2 - \eta_2 \rho_2 g_i - \frac{\alpha}{V}\int_{A_{12}} C n_i dA \tag{44}$$

where the term $(1/V)(\bar{p}_2 - \bar{p}_1)\int_{A_{12}} n_i dA$ is assumed to be small and omitted. Combining Eq. 44 with the flow Eq. 32 arrived at previously we can eliminate $\partial_i \bar{p}_1$, obtaining

$$\mu_1(b_1 + d_1)\left(\frac{q_i^{(2)}}{\eta_2} - \frac{q_i^{(1)}}{\eta_1}\right) = -\eta_2(\partial_i \bar{p}_2 - \rho_2 g_i) + \frac{\alpha}{V}\int_{A_{12}} C n_i dA \tag{45}$$

as the second flow equation. If we now define a capillary vector

$$\vec{N}_c \equiv \frac{\mu_1 \vec{q}^{(1)}}{\alpha} \tag{46}$$

which has as its magnitude the capillary number $\mu_1 q^{(1)}/\alpha$ then we may rewrite Eq. 45 as

$$\frac{\mu_1(b_1 + d_1)}{\eta_2^2}\vec{q}^{(2)} = -\nabla\bar{p}_2 + \rho_2\vec{g} + \frac{\alpha}{\eta_2}\left[\frac{b_1 + d_1}{\eta_1}\vec{N}_c + \frac{1}{V}\int_{A_{12}} C\vec{n} dA\right]. \tag{47}$$

Equations 32 and 47 or 45 together are the flow equations for a continuous wetting phase (fluid 1) and a partially mobilized discontinuous nonwetting phase (fluid 2) in a porous medium. Here the porous medium is assumed to be completely coated by the wetting phase. In the limit as the flow of the discontinuous phase goes to zero, Eq. 32 becomes Darcy's equation for the continuous phase. Furthermore the righthand side of Eq. 47 then yields a criterion for the mobilization of the discontinuous phase. Here if the magnitude of the terms attempting to mobilize the droplet are greater than those impeding its progress the blob is mobilized. Thus

$$\left|\frac{b_1 + d_1}{\eta_1} N_{ci} - \eta_2 \frac{(\partial_i \bar{p}_2 - \rho_2 g_i)}{\alpha}\right| > \left|\frac{1}{V}\int_{A_{12}} C n_i dA\right| \qquad i = x,y,z \tag{48}$$

is the criterion for blob mobilization.

Equation 48 provides physical insight into the mechanisms controlling the mobilization of oil ganglia; however, efforts to further reduce the righthand side are clearly required before direct practical use of this equation can be envisaged. Such reductions, though not readily apparent in the generality considered in this paper, may be possible in particular cases through experimental investigations.

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NOTATION

a_1	= Coefficient defined in Eq. 28
A_1	= collection if interfaces (in V) involving phase 1
A_{12}	= phase 1–phase 2 interfaces
A_{1s}	= phase 1–solid interfaces
b_1	= coefficient defined in Eq. 29
C	= curvature of an interface
C^*	= curvature of flood front
c_1	= coefficient defined in Eq. 30
d_1	= coefficient defined in Eq. 31
dA	= area of surface element
dA_i	= $n_i dA$
$g_i, \vec{g}$	= gravitational acceleration
k	= absolute permeability
k_1	= relative permeability of phase 1

$n_i, \vec{n}$	= unit normal to a surface element
$\vec{N}_c$	= capillary vector; cf. Eq. 46
N_c	= capillary number, $\|\vec{N}_c\|$
p	= pressure
$p_\alpha, \alpha = 1,2$	= pressure in phase α
p_c	= capillary pressure
p_o	= integration constant; cf. Eq. 21
$q_i, \vec{q}$	= filter velocity
$\vec{q}^{(\alpha)}, \vec{q}_\alpha$	= filter velocity of phase $\alpha = 1,2$
$s_\alpha, \alpha = 1,2$	= saturation of phase α
s_{or}	= residual oil saturation
$\vec{u}$	= velocity of interface
$\vec{v}, v_i$	= fluid velocity
$\vec{v}^{(\alpha)}, \vec{v}_\alpha$	= fluid velocity of phase $\alpha = 1,2$
V	= volume of regions used in volume averaging; cf. Eq. 1

Greek Letters

α	= interfacial tension
α^*	= effective interfacial tension
$\partial_i, i=x,y,z$	= partial derivatives
η	= porosity
η_α	= fractional volume occupied by phase $\alpha = 1,2$
π_{ik}	= momentum flux density tensor
ρ	= mass density
σ_{ik}	= viscosity stress tensor
μ	= viscosity

Subscripts and Superscripts

$\alpha, (\alpha) = 1,2$	= phase α
$i,j,k = x,y,z$	= cartesian components

Special Symbols

$< >$	= average over V; cf. Eq. 1
$\overline{(\ )}$	= occupied by the phase; cf. Eq. 2

LITERATURE CITED

Chuoke, R. L., P. van Meurs, and C. von der Poel, "The instability of slow, immiscible, viscous liquid-liquid displacements in permeable media," *Trans. Am. Inst. Min. Eng.*, **216**, 188 (1959).

Clampitt, R. L., and P. D. Fleming, "Research digs into micellar flooding processes," *Oil and Gas J.* (Jan. 14, 1980).

Hubbard, M. K., "Darcy's law and the field equations of the flow of underground fluids," *A.I.M.E. Petrol. Trans.*, **207**, 222 (1956).

Larson, R. G., L. E. Scriven, and H. T. Davis, "Percolation theory of residual phases in porous media," *Nature*, **268**, 409 (1977).

Melrose, J. R., and C. F. Brandner, "Role of capillary forces in determining microscopic displacement efficiency for oil recovery by water-flooding," *J. Can. Pet. Tech.*, **13**, 54 (1974).

Newman, S. P., "Theoretical derivation of Darcy's Law," *Acta. Mech.*, **25**, 153 (1977).

Ng, K. M., H. T. Davis, and L. E. Scriven, "Visualization of blob mechanics in flow through porous media," *Chem. Eng. Sci.*, **33**, 1009 (1978).

Ng, K. M., and A. C. Payatakes, "Stochastic simulation of the motion, breakup and stranding of oil ganglia in water-wet granular porous media during immiscible displacement," *AIChE J.*, **26**, 419 (1980).

Patel, J. G., M. G. Hedge, and J. C. Slattery, "Further discussion of two-phase flow in porous media," *AIChE J.*, **18**, 1062 (1972).

Pathak, P., P. H. Winterfeld, H. T. Davis, and L. E. Scriven, "Rock structure and transport therein: unifying with Voroni models and percolation concepts," First Joint SPE/DOE Symp. on Enhanced Oil Recovery, Tulsa, OK (April 20–23, 1980).

Payatakes, A. C., K. M. Ng, and R. W. Flumerfelt, "Oil ganglion dynamics during immiscible discplacement: model formulation," *AIChE J.*, **26**, 430 (1980).

Slattery, J. C., "Flow of viscoelastic fluids through porous media," *AIChE J.*, **13**, 1066 (1967).

Slattery, J. C., "Single-phase flow through porous media," *AIChE J.*, **15**, 866 (1969).

Slattery, J. C., "Two-phase flow through porous media," *AIChE J.*, **16**, 345 (1970).

Spanos, T. J. T., "The surface conditions for viscous-displacement in a homogeneous porous medium," *Can. J. Phys.*, **57**, 1738 (1979).

Whitaker, S., "The equations of motion in porous media," *Chem. Eng. Sci.*, **21**, 291 (1966).

Whitaker, S., "Advances in the theory of fluid motion in porous media," *Ind. Eng. Chem.*, **61**, 14 (1969).

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Simplification of Quasilinearization Method for Parameter Estimation

An alternate development of the quasilinearization method for parameter estimation is presented to enable a more efficient implementation of the algorithm. Similarity of this algorithm to Gauss-Newton method is shown and attention is given to systems having a nonlinear relationship between the observed and state variables. To overcome the problem of a small region of convergence, the use of direct search optimization is proposed for the first few iterations, followed by the simplified quasilinearization algorithm.

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SCOPE

Off-line parameter estimation in systems described by sets of ordinary differential equations is very important in process modelling, simulation and optimization. Among the most common methods are the quasilinearization method (Bellman and Kalaba, 1965; Lee, 1968), the Gauss-Newton method (Bard, 1970, 1974), and Marquardt's modification of Gauss-Newton method (Marquardt, 1963; Bard, 1970).

Quasilinearization is best known for its fast quadratic convergence to the optimum, but a major problem is its small region of convergence (Seinfeld and Gavalas, 1970; Seinfeld and La-